**Probability – Notes**

**Last year, Malcolm was late to school on average 5 days out of 100 days. Write mathematical expressions (but don’t evaluate) for the probability that in a school week of 5 days, Malcolm is:**

**[a] Late only on the first day.**

0.05 x 0.954

**[b] Late on the first 3 days.**

0.05

1

1

0.05

0.05

\_\_\_\_\_ x \_\_\_\_\_ x \_\_\_\_\_ x \_\_\_\_\_ x \_\_\_\_\_ = 0.053

**[c] Late only on the first 3 days.**

0.053 x 0.952

**[d] Late only on exactly 3 days.**

5C3 x 0.053 x 0.952

**[e] Late on at least 3 days.**

5C3 x 0.053 x 0.952 + 5C4 x 0.054 0.95 + 0.055

**[f] Late only on the first and the fifth day given that he was late on exactly 2 days in the school week.**

$\frac{0.05^{2} x 0.95^{3}}{5C\_{2} x 0.05^{2} x 0.95^{3}}$ = $\frac{1}{5C\_{2}}$

**The Collett Boat Company has a fleet of 3 boats. From Company records for the last 2 years, the Jupiter is chosen by 55% of customers, the Venus by 28% of customers and the Mars by the remaining customers. The probabilities that each boat breaks down during a 2-hour trip are Jupiter 0.2; Venus 0.15; Mars 0.3.**

**[a] If all 3 boats are out on hire for a 2-hour trip, find the probability that:**

**(i) None breaks down.**

0.8 x 0.85 x 0.7 = 0.476

**(ii) The Mars and one other boat in this fleet breaks down.**

P(Only Mars and Jupiter) + P(Only Mars and Venus) = 0.3 x 0.15 x 0.8 + 0.3 x 0.2 x 0.85 = 0.087

**[b] Only one boat is out on hire for a 2-hour trip. What’s the probability that it will break down?**

0.55 x 0.2 + 0.28 x 0.15 + 0.17 x 0.3 = 0.203

**[c] News comes through that the one boat out on hire has broken down. What’s the probability that it’s the Jupiter?**

P(Jupiter | one boat broken down) = $\frac{P(Jupiter ∩ one boat broken down)}{P(One boat broken down)}$ = $\frac{0.55 x 0.2}{0.203}$ = 0.5419

**At a certain airport, the probability that a plane takes off on time given that weather conditions are fine is 0.9. The probability that a plane takes off on time given the weather conditions are bad is 0.7. The probability of weather conditions being fine or the plane taking off on time is 0.955.**

**[a] Find the probability of a plane taking off on time in fine weather conditions.**

0.9

On time

Fine

0.1

Late

0.7

On time

Bad

0.3

Late

P(Fine ∩ on time) + P(Fine ∩ late) + P(Bad ∩ on time) = 0.955

0.9x + 0.1x + 0.7(1–x) = 0.955

P(Fine) = 0.85

P(Fine ∩ on time) = 0.85 x 0.9 = 0.765

**[b] Find the probability of weather conditions being fine given that a plane took off on time.**

P(Fine | on time) = $\frac{P(Fine ∩ on time)}{P(On time)}$ = $\frac{0.765}{0.765+0.15x0.7}$ = 0.8793

**Given that P(A) = 0.5, P(B) = 0.8 and P(Ā ∩ B̄) = 0.05 and that the events A and B are independent, determine if these results are consistent with the rules of probability.**

P(A ∪ B) = 1 – 0.05 = 0.95

P(A ∩ B) = 0.5 + 0.8 – 0.95 = 0.35

P(A) x P(B) = 0.4 ≠ 0.35

P(A ∩ B) ≠ P(A) x P(B) → not consistent.

**Given that P(A’) =** $\frac{3}{10}$**, P(B|A) =** $\frac{2}{7}$ **and P(A|B) =** $\frac{1}{2}$**:**

**[a] Find P(B).**

P(A) = 1 – $\frac{3}{10}$ = $\frac{7}{10}$

P(B|A) = $\frac{P\left(A ∩ B\right)}{P(A)}$ = $\frac{P(A ∩ B)}{\frac{7}{10}}$ = $\frac{10 x P(A ∩ B)}{7}$ = $\frac{2}{7}$ → P(A ∩ B) = $\frac{1}{5}$

P(A|B) = $\frac{0.2}{P(B)}$ = $\frac{1}{2}$ → P(B) = 0.4

**[b] Find P(A’ ∩ B’).**

P(A ∪ B) = $\frac{7}{10}$ + $\frac{4}{10}$ – $\frac{1}{5}$ = 0.9

P(A’ ∩ B’) = 1 – 0.9 = 0.1

**[c] Determine with reasons if A and B are independent.**

P(B) = 0.4

P(B|A) = $\frac{2}{7}$

P(B) ≠ P(B|A) → not independent.

**Chin either drives to work or takes a train to work. The probability that he’s on time for work is 0.86. The probability that he’s late for work given that he drives to work is 0.3. The probability that he’s on time for work given that he take s a train to work is 0.9.**

**[a] Find the probability that he’s on time for work given that he drives to work.**

1 – P(on time|drive) = 1 – 0.3 = 0.7

**[b] Find the probability that he drives to work.**

1 – x

x

0.9

0.1

0.3

0.7

Drive

Late

On time

Late

On time

Train

0.86 = 0.9x + 0.7(1 – x) = 0.2x + 0.7 → x = 0.8

P(drive) = 1 – 0.8 = 0.2

**[c] Given that he’s late for work, what’s the probability that he took the train to work?**

P(late ∩ train) = 0.8 x 0.1 = 0.08

P(train|late) = $\frac{P(train ∩ late)}{P(late)}$ = $\frac{0.08}{0.2 x 0.3 + 0.8 x 0.01}$ = $\frac{4}{7}$

**A red box has 4 books and blue box has 8 books. All books are different. A total of 5 books are chosen from these 2 boxes.**

**[a] In how many ways can this be done?**

12C5 = 792

**[b] What’s the probability that all the books from the red box are chosen?**

(4C4 x 8C1)/(12C5) = $\frac{1}{99}$

**[c] What’s the probability that at least one of the books chosen is from the red box?**

P(none) = (4C0 x 8C5)/12C5 = $\frac{7}{99}$

P(at least one) = 1 – $\frac{7}{99}$ = $\frac{92}{99}$

**[d] What’s the probability that more books from the red box are chosen?**

3, 2 4, 1

(4C3 x 8C2 + 4C4 x 8C1)/12C5 = $\frac{5}{33}$

**Consider the digits 0 to 9 inclusive and all the letters of the English Roman alphabet. 12 characters consisting of digits and letters are chosen.**

**[a] What’s the probability that all the characters chosen are letters?**

(26C12 x 10C0)/36C12 = 0.0077

**[b] What’s the probability that all the digits are chosen?**

(10C10 x 26C2)/36C12 = 0.00000026

**[c] Given that all the characters chosen are letters, what’s the probability that all the vowels are chosen?**

(5C5 x 21C7)/26C12 = 0.012

**[d] What’s the probability that no vowels or even digits were chosen?**

(5C0 x 5C0 x 26C12)/36C12 = 0.0077

**[e] What’s the probability that at least one vowel or even digit was chosen?**

1 – (5C0 x 5C0 x 26C12)/36C12 = 0.99

**Given that P(A) = 0.4, P(C|A) = 0.3 and P(C|Ā) = 0.2, find P(A|C).**

$\frac{P(A ∩ C)}{0.4}$ = 0.3 → P(A ∩ C) = 0.12

P(Ā) = 1 – P(A) = 1 – 0.4 = 0.6

$\frac{P(C ∩ Ā)}{0.6}$ = 0.2 → P(C ∩ Ā) = 0.12

P(C) = P(C ∩ Ā) + P(A ∩ C) = 0.12 + 0.12 = 0.24

$\frac{P(A ∩ C)}{P(C)}$ = $\frac{0.12}{0.24}$ = 0.5

**England and Germany play a series of 2 soccer matches. Each match doesn’t end in a draw. The probability that Germany will win both matches is 0.42 and the probability that Germany will lose both matches is 0.125. The probability that Germany will win the first match is 0.7.**

0.28

0.4

0.6

0.3

0.42

England

Germany

Germany

England

England

Germany

0.7

0.165

0.55

0.135

0.45

**[a] Find the probability that England wins exactly one of the matches.**

P(England wins M1 ∩ England loses M2) = 0.165

P(England loses M1 ∩ England wins M2) = 0.28

0.165 + 0.28 = 0.445

**[b] Find the probability that England win the second match.**

P(England loses M1 ∩ England wins M2) = 0.28

P(England wins M1 ∩ England wins M2) = 0.135

0.28 + 0.135 = 0.415

**[c] Find the probability that Germany lose the first match given that they won the second match.**

P(Germany loses M1 ∩ Germany wins M2) = 0.165

P(Germany wins M2) = 0.165 + 0.42 = 0.585

$\frac{0.165}{0.165+0.42}$ = 0.2821

**[d] Determine with reasons if the events that Germany win match one and the even that Germany win match 2 are independent.**

P(Germany wins M1 ∩ Germany wins M2) = 0.42

P(Germany wins M1) x P(Germany wins M2) = 0.7 x (1 – 0.415) = 0.4095

Therefore the 2 events aren’t independent.